

# Post-disaster volatility of blood donations in an unsteady blood supply chain

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## Abstract

The stochastic behaviour of both transfusion (demand) and blood donations (collection) is a challenge for the blood supply chain. Although donations are not fully within the control of blood supply chain, the blood service can marginally moderate it by postponing appointments in the case of having an overstock, or by triggering a call for additional blood when faced with shortages. Such shortages are often observed as a consequence of catastrophic events. Past studies show that the response to a call for blood after a disaster is substantive. Yet the consequential impact on the supply chain is not well understood. This is due to the perishability of blood and the fact that donors are not eligible to give blood for a certain period after a donation has been made. In this study the donation process is modelled with a Markov chain and the impact of a call for blood resulting from a disaster is investigated. This paper highlights new actionable insights that aid planners to mitigate the negative impacts of a substantial response to a call for blood.

## KEYWORDS

Blood supply chain; Donation; Disaster relief operations; Call for blood; Perishable inventory; Humanitarian aid supply chain.

## 1. Introduction

For numerous patients requiring transfusions a reliable population of blood donors is a necessity. In Australia, 1 in 30 people currently give blood; however, 1 in 3 people will need blood in their lifetime. In order to reach a steady state, the process relies

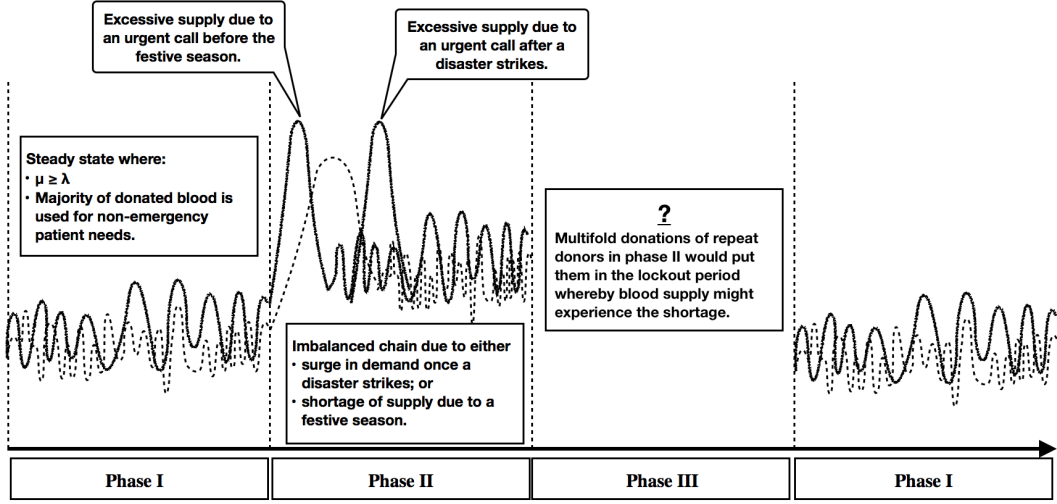
on repeat donors. In Australia, repeat donors supplied 93% of all collected blood in 2016 ([ARCBS and Kirby Institute 2017](#)). This routine (supply-demand equilibrium) can be continued until a disaster strikes or an urgent call is issued prior to a festive season. These additional donations ensure sufficient supply of blood, addressing the expected surge in demand. Such an urgent call is able to resolve the surge in demand for blood given that sufficient number of donors respond. However, the subsequent impact of these additional donations on the continuity of blood supply chain requires careful attention. Firstly, once a disaster strikes, a significant portion of the donated blood is used to treat the affected people in addition to background levels of demand. Secondly, the supply of blood might experience disruption due to the ineligibility of repeat donors in donating blood in the near future.

Blood products are perishable. RBCs have a shelf life of 35 to 42 days depending on the government regulations ([Pierskalla 2004](#)), platelets have a shelf life of 5 days and plasma can be stored for up to one year ([Osorio, Brailsford, and Smith 2015](#)). Processing and testing time also decrease the shelf life of blood products taking about two days before the blood products are ready for delivery to hospitals ([Abbasi, Vakili, and Chesneau 2017](#)). This processing and testing time effectively makes the practical shelf life of RBCs and platelets 40 days and 3 days respectively. Due to perishability of RBCs and platelets, over-collected blood in some periods cannot effectively ensure the supply and demand equilibrium in future periods with surged demand or declined supply. Often the reaction of donors to a catastrophic event exceeds the amount needed due to insufficient or uncertain information on the nature of the catastrophe, and thus the supply might be significantly greater than the actual demand. For example, during the black Saturday bushfires in Victoria, Australia, an urgent call for additional blood significantly increased the quantity of donations. About 40,000 people registered online in 10 days, a tremendous increase over the usual average of 2200 registrations per month. However, the actual need for blood was significantly less than predicted, causing many units to outdate. Only 18% of the collected blood was used for burns sufferers and just over 30% used for non-related disaster needs ([Australian Red Cross 2009](#)). Although not specifically reported, this implies that about 50% of the blood collected in response to this disaster was discarded.

An often overlooked fact in modelling the blood supply chain is that the donor population is limited in both size and donation frequency (Osorio, Brailsford, and Smith 2018). Most countries rely on a relatively small population of donors to meet demand. Further, once a donation has been given, the donor is prohibited from making another for a fixed period of  $T$  days. Donation frequency is also an issue; some donors give blood on a regular basis, others less so and some may donate less than 5 times in their lives. If the inventory at blood service is greater than the desired level, the blood service reduces the quantity of donations and in case of a significant excess of inventory, the blood service may export extra units or even quarantine and outdated them early in order to reduce the age of units issued to hospitals.

In Figure 1 the response of the supply-demand equilibrium of a blood supply chain to a disaster is categorised into three phases. In phase I or pre-disaster stage, the blood supply chain is in steady state whereby the supply rate ( $\mu$ ) is greater than or equal to the consumption rate ( $\lambda$ ) and blood usage remains at background levels. When the blood bank inventory is less than the desired level, it is usual to apply restrictions when fulfilling hospital orders. This ensures that available blood goes to those hospitals where it is most needed. In phase II where a disaster strikes, the blood service experiences a surge in demand that cannot be satisfied from the inventory on hand and therefore might request an increase in donations. Sometimes, an urgent call for blood is also placed prior to a festive season as a decline in blood supply is expected and demand might be higher due to the possible increased rate of accidents. Depending on the resilience of a blood supply chain in phase III, it will eventually return to its original steady state (phase I).

Since imbalance in the blood supply chain can potentially have a severe impact on patient welfare, prudent management of the supply chain is a critical challenge (Pierskalla 2004; Dobbin, Wilding, and Cotton 2009; Stanger 2013). The increase in wastage and the extra cost of collection in response to a disaster is well understood, but the associated negative impact of reducing the population of eligible donors in a period after an urgent call for blood has not been studied. In other words, given that donors who donated blood in phase II are unable to give blood during the lockdown period (e.g. 84 or 56 days), the supply chain might experience a supply shortage over



**Figure 1.** There is a knowledge gap regarding how the  $T$ -day lockout of repeat donors after a significant increase in blood donations might impact the continuity of blood supply (donation: solid line; demand: dash line).

phase III. This study explores the impact of an urgent call for blood on the availability of repeat donors and on the resilience of the blood supply chain. This paper contributes to the body of knowledge in the blood supply chain field in two ways. First, it offers a novel methodology to model blood donations by repeat donors providing actionable insights in blood donor management polices. Second, it highlights the impact of an urgent call for blood after a disaster and quantifies the expected decline in blood donations by repeat donors in a certain period afterwards. Third, it provides some policy recommendations to the blood services on how to prevent such a decline in blood donations in phase III.

The rest of the paper is organized as follows. Section 2 reviews the related literature. Section 3 formulates the blood donation frequency of the repeat donors. Section 4 quantifies the fall in repeat donors' donations after an urgent call for blood. Computational experiments and sensitivity analysis are presented in Section 5. Section 6 deliberates on the policy implications of our analysis on urgent call for blood actions. Section 7 presents a real case study and finally Section 8 offers conclusions.

## 2. Literature Review

Managing a blood supply chain involves effective and efficient control of the flow of blood and blood products from the donor to the patient in order to minimise the total blood supply chain cost, maintain or improve the service levels, and to minimise the risk of shortage and outdates. This study classifies the literature into two streams: the steady state (phase I) and the response to an urgent call for blood (phases II and III).

### 2.1. *Blood Supply Chain in Steady State (phase I)*

From an operations research perspective, many studies have addressed the the steady state optimisation of various echelons of the blood supply network. This section of blood supply chain literature can be categorised at three different hierarchical levels: individual hospital level, regional blood centre level, and supply chain level. Some studies have focused on only one of these hierarchical levels, while others have looked into the problems across multiple levels. Studies that targeted the individual hospital level generally examined hospital blood blanks. Various problems were solved in this regard such as evaluating the factors affecting stock level and reducing the blood wastage in hospital blood banks (Dumas and Rabinowitz 1977; Pink, Thomson, and Wylie 1994; Perera et al. 2009; Yates et al. 2017); developing benchmarking targets for red blood cell outdates at hospital blood banks (Heddle et al. 2009); simulating the blood inventory system to reconsider the blood ordering strategy or improve the inventory policy (Rabinowitz 1973; Vrat and Khan 1976; Lowalekar and Ravichandran 2017); and examining how alternative hospital policies might impact the blood inventory policy (Pegels et al. 1977).

In order to address shortages and outdates of blood, coordination between hospitals and blood banks have gained special attention among researchers (Beliën and Forcé (2012)). This covers such ideas as allocation policies of a perishable product from a regional centre to different locations in the region (Kendall 1980; Federgruen, Prastacos, and Zipkin 1986; Tetteh 2008), discussing a redistribution system for near-outdate red blood cells to transport units that are about to outdate from a low-usage hospital to a high-usage hospital (Denesiuk et al. 2006), collections planning for regional

blood centres to smooth seasonal variability of supply and demand for blood (Cumming et al. 1976), developing a blood inventory level forecast system to alert regional blood centres (Frankfurter, Kendall, and Pegels 1974), investigating various issuing policies implemented in regional blood banks with uncontrollable replenishment (Abbasi and Hosseinifard 2014) and lastly papers dealing with transshipment and the issue of whether a centralised system outperforms a decentralised system (Hosseinifard and Abbasi 2018; Dehghani and Abbasi 2018).

A holistic view has the aim of improving the whole blood supply chain rather than tackling a problem regarding a single level within the chain. This is because the system dynamics of interaction of the various levels within the supply chain can produce variation over and above that attributed to stochastic supply and demand (Clay et al. 2018). Decision support systems to ameliorate interregional policies would fall also under this category. Like any other supply chain, researchers have tried to maximise throughput by end-to-end optimisation of the chain; for example, building a simulation model to assess the entire chain (Rytilä and Spens 2006; Abbasi, Vakili, and Chesneau 2017); emphasising on the necessity of effective collaboration among blood centres and hospital transfusion services (Fontaine et al. 2009); and evaluating the performance of the entire blood supply chain (Katsaliaki 2008).

## ***2.2. Blood Supply Chain in Unsteady State (Phases II and III)***

A resilient blood supply requires that a sufficient quantity of donors are available to give blood. It is estimated that more than half of the population are eligible to donate blood (Martín-Santana and Beerli-Palacio 2012). However, despite the various awareness campaigns only 5% actually do (France, France, and Himawan 2007; Lacetera and Macis 2010). Within this population subset, only 1% donate regularly (Ringwald, Zimmermann, and Eckstein 2010). Further, given that over half of all new blood donors fail to donate a second time (Thomson et al. 1998; Ownby et al. 1999; France et al. 2004), it is becoming increasingly challenging to retain first-time donors (Wu et al. 2001). It is recognised that majority of the nation's blood supply is obtained by a relatively small number of committed individuals who donate regularly (Royse and Doochin 2003; Thomson et al. 1998). Repeat donors represent between

78% and 91% of the total blood donated (Gillespie and Hillyer 2002; Martín-Santana and Beerli-Palacio 2012). The optimal strategy to ensure stability of the blood supply is to improve retention of first-time donors and to enhance the frequency of donations within the repeat donor pool (Godin et al. 2007). In fact, it is feasible to achieve a significant surge in blood donations with a relatively small increase in the number of donations from repeat donors (Ringwald, Zimmermann, and Eckstein 2010). Consequently, special attention should be paid to donor retention as the cost of recruiting new donors is higher than that of retaining repeat donors. Further, repeat donors represent a safe and reliable blood supply (Masser et al. 2009). Therefore, to achieve a stable blood supply chain, it seems necessary to appraise any parameter which might impact on the donor supply pool with a special attention being given to the volume of repeat donors.

Optimisation of all aspects of demand and supply may result in a balanced and steady blood supply chain, however, a disaster or a catastrophic event might interrupt this equilibrium. It is often observed that the donation rate in response to a disaster increases. Busch et al. (1991) showed that the donation rate doubled in response to the urgent call for blood after the 1989 San Francisco earthquake. Glynn et al. (2003) showed that the urgent call for blood after the September 11 disaster increased the number of donations by 2.5 times in the week after the disaster when compared to the week before the disaster. This study also reported that number of first-time donors was 5.2 times greater and the number of donations from repeat donors was 1.5 times greater. Kidder (2010) reported that less than 1% of the new donors after the September 11 disaster returned to give blood. A similar observation was made in Tran et al. (2010) where the commitment of first time donors that become repeat donors after a national emergency was investigated. The authors concluded that new donors after a disaster are less likely to become repeat donors. Notwithstanding this, the overwhelming response to the urgent call for blood after the September 11 disaster cost nearly \$5 million due to the collection and processing of blood that was unusable. Additionally, a threefold increase in the number of units discarded due to outdates was reported.

Similar observations have been made in other countries. Liu et al. (2010) reported

that a call for blood after an earthquake in Sichuan province in China resulted in a 68% increase in blood collection with an 89.5% and 34% increase observed in first time and repeat donors respectively. The usual number of repeat donors was on average 265 per week. In the first week after the disaster, it increased to 355. Repeat donors made approximately 72% of these donations. [Kasraian \(2010\)](#) showed that the number of first time donors increased by a multiple of 13.8 and the number of donations from repeat donors increased to 5.9 times the usual rate after the Bam earthquake in Iran. The proportion of repeat donors before the disaster was 85.3% and it was 71.4% after the disaster. [Abolghasemi et al. \(2008\)](#) reported that after the Bam earthquake, 108,985 units were collected while only 21,347 units (23%) were actually used. As a result, it can be concluded that an urgent call for blood leads to a significant increase in blood supplied (phase II in Figure 1). It is also clear that increased spending on awareness campaigns results in a greater supply. Given the significant cost associated with storing and disposing of excess blood, it is worth reviewing the accuracy of the proposed models with respect to estimating the post-incident demand.

A few studies have tried to estimate the required quantity of blood needed in response to a disaster. They confirm the need for innovative design and planning of blood supply chains that function effectively during and after disasters. However, the unique characteristics of the network structure including the dependency of demand on multiple externalities such as disaster severity makes the design of a blood supply chain extremely complex ([Pierskalla 2004](#); [Sheu 2007](#); [Nagurney et al. 2013](#)). Whilst the inadequate supply of blood during catastrophic incidents can have implications such as increased mortality rate ([Beliën and Forcé 2012](#)), the inaccuracy of demand forecasts may also impose substantial costs to blood facilities. Unpredictable demand with high variability during different phases of a disaster would also increase this complexity. Further, [Tabatabaie et al. \(2010\)](#) highlighted that demand is highest during the first 24 hours of a disaster. Other concerns relate to the restriction on storage and transportation of blood products. For instance, blood products have tight expiration dates and should be stored in a specific storage temperature range ([Delen et al. 2011](#)). A shortage of appropriate storage elevates the risk of post-disaster blood wastage when blood collection centres face a large influx of donations within the early days of a catastrophic



event ([Kuruppu 2010](#)). Other efforts have been made to design scenarios to estimate the required blood units and to plan for collection in the aftermath of an emergency thereby mitigating the possibility of blood over-collection. For example, [Tabatabaie et al. \(2010\)](#) and [Guo et al. \(2012\)](#) concluded that post-earthquake first-time donors are less likely to donate again in the absence of continuing motivation strategies. [Kuruppu \(2010\)](#) indicated the over-collection concerns in disaster events by reviewing Tsunami disaster in 2004 in Sri Lanka. It is clear that over-collection of blood causes significant loss due to the cost of expired blood and wasted efforts in collection, processing, packaging, and awareness campaigns. On top of economic losses, this research investigates the other possible downside of the over-collection which occurs due to the unavailability of repeat donors in phase III. Various mathematical models have been used to address network design problem (location of collection centres and movement of blood units from centres to demand points) of a blood supply chain in disasters (for examples refer to [Fahimnia et al. \(2017\)](#), [Jabbarzadeh, Fahimnia, and Seuring \(2014\)](#) and [\(Zahiri and Pishvae 2017\)](#)).

The previous studies demonstrate that the community response to an urgent call for blood in response to an emergency causes a significant surge in blood collection. Moreover, this overwhelming response leads to millions dollars being lost as a result of using resources for collection and processing the blood units and discarding them later due to expiry. There is however, another, longer term impact of the overwhelming response to an urgent call for blood on the blood supply chain that has not been studied or understood: that is the shortage in collection after a period of disaster (phase III). This is due to unavailability of repeat donors that gave blood during the emergency (phase II) but are prohibited from donating again until their lockout period is complete.

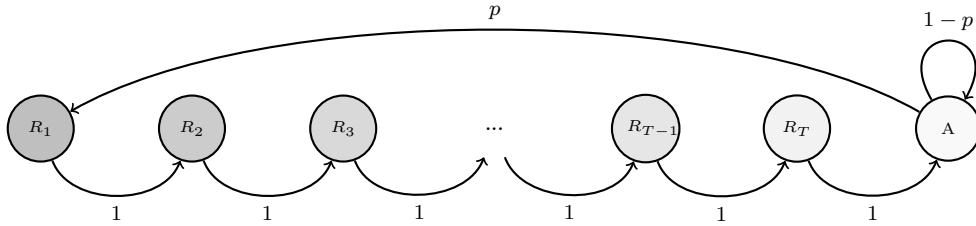
In summary, the mainstream literature regarding the unsteady state of blood supply chain provides analysis of donations in response to calls for blood to satisfy the increased demand over the duration of the disaster. The post disaster impact of an urgent call on the continuity of blood supply chain is still unclear. In this paper, the impact of an overwhelming response to an urgent call for blood on the decline in subsequent collections and its consequential impact on the blood supply chain is

studied.

### 3. Modelling of Blood Donations by Repeat Donors

The blood donation pool has two main sources: first time donors and repeat donors. When an urgent call for blood arises, collection from both first time donors and repeat donors increases. After donation, both of these donor sources are prevented from donating again for  $T$  days. This is not an issue for first time donors as these rarely go on to become repeat donors. However, the repeat donor pool is significantly impacted by this lockout period and this causes a decline in future collections. Consequently, this study focuses on the impact of a decline in the repeat donor population after an urgent call for additional blood and the corresponding reduction in donations in phase III.

Repeat donors, when they are eligible, are assumed to donate blood regularly with a fixed probability,  $p$ , noting that  $p \in [0, 1]$  is not same for all donors and it is called recurrence probability. It is assumed that the repeat donor population comprises of  $N$  donors and  $p$  has a probability distribution. We formulate the transition of repeat donors from being available (state A) to being in one of the lockout states (rest (R) states) via a discrete time Markov process. Figure 3 shows the structure of the collection process from repeat donors:



**Figure 2.** Illustration of the blood donations process for repeat donors as a discrete time Markov chain (the values on arcs show the transition probability).

In line with actual practice, donors will spend  $T$  days (e.g 84 days or 56 days depending on the regulation of the blood service that may vary in each country) in lockout before being eligible to make another donation. For example, state  $R_5$  in Figure 3 means that the donor gave blood 5 days ago and will not be available for donation

for another  $T - 5$  days. The transition matrix of the discrete time Markov chain is presented below:

$$\begin{array}{c}
 R_1 \quad R_2 \quad R_3 \quad \dots \quad R_T \quad A \\
 \begin{array}{c} R_1 \\ R_2 \\ R_3 \\ \vdots \\ R_T \\ A \end{array} \begin{pmatrix} 0 & 1 & 0 & \dots & 0 & 0 \\ 0 & 0 & 1 & \dots & 0 & 0 \\ 0 & 0 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 1 \\ p & 0 & 0 & 0 & 0 & 1 - p \end{pmatrix}
 \end{array}$$

An intuitive interpretation of expected probabilities for the stationary distribution of each Markov chain node is that the flow of probability into each state must equal the flow of probability out of each state (Winston 2004, p. 193). Applying this theory to the inflow and outflow of equilibrium probabilities for a donor being available (being in state A) results in:

$$\pi_A(1 - p) + \pi_{R_T} = \pi_A \quad (1)$$

where  $\pi_j$  is the probability of being at state  $j$  for  $j \in \{1, 2, \dots, T, A\}$ , at any given time. Applying the same approach on the rest states results in  $\pi_{R_1} = \pi_{R_2} = \dots = \pi_{R_T}$ . In addition, the sum of all these probabilities is one. Hence:

$$\sum_{j=1}^T \pi_{R_j} + \pi_A = T\pi_{R_T} + \pi_A = 1 \quad (2)$$

Substituting Equation 2 in the Equation 1 results in:

$$\pi_A(1 - p) + \frac{1 - \pi_A}{T} = \pi_A$$

Thus:

$$\pi_A = \frac{1}{1 + Tp}$$

Let  $X$  be the event that a repeat donor gives blood on a given day. Since  $\pi_A$  is the probability that a repeat donor is *available* (not in the lockout period), and  $p$  is the *recurrence* probability, the probability that a repeat donor donates blood in a given day is  $\pi_A \times p$ , therefore:

$$\Pr(X) = \frac{p}{1 + Tp} \tag{3}$$

It is clear from Equation 3 that  $X \sim \text{Bern}(p)$ . Hereafter, the *recurrence* probability of donor  $i$  is denoted by  $p_i$ . Recall that  $N$  is the size of the population of repeat donors and let  $L$  be the number of repeat donors that donate on a given day. We let  $N$  to be a relatively large and fixed number. It is a reasonable assumption as to establish a supply-demand equilibrium, blood supply chains require a large panel of repeat donors maintained by recruiting new repeat donors to replace the retired donors. The justification behind the selected value of  $N = 100,000$  in the ‘computational result’ section is that in Australia there are approximately 458,132 repeat donors ([ARCBS and Kirby Institute 2017](#)). Considering the population of Australia, around 2% of Australian population are repeat donors. Given the population of State of Victoria is about 5 million people, it is expected that there are 100,000 repeat donors. We also assume that repeat donors are eligible for blood donations after the lockout period, however, in some circumstances such as illness due to some infections, repeat donors are ineligible for blood donations for some periods that are not considered in

our model. Furthermore, we assume the donation by a repeat donor when he/she is eligible (available) to donate is independent of the donations of other repeat donors and is a Bernoulli trial with a specific probability of donation (*success*) that might be different from other repeat donors. Hence, the number of donations by the repeat donors in a given day ( $L$ ) follows a Poisson Binomial distribution defined as the sum of  $n$  independent Bernoulli trials with not necessarily identically distributed success probabilities of  $p_1, p_2, p_3, \dots, p_n$ .

As the expected value of the Poisson Binomial distribution is estimated as  $\sum_i p_i$ , conditioning on  $p_i$ , the expected value of  $L$  can be obtained as:

$$\mathbb{E}(L \mid p_1, \dots, p_N) = \sum_{i=1}^N \frac{p_i}{1 + T p_i}$$

From a planning perspective, it is necessary to estimate both the mean of number of donations and the variability of this parameter. In Proposition 1, we derive the variance of  $L$ .

**Proposition 1.** The variance of number of donations from repeat donors with total population of  $N$  is obtained as (Proof in Appendix A):

$$Var(L) = \mathbb{E}(L) - \frac{\mathbb{E}(L)^2}{N} \quad (4)$$

The variance formula in Equation 4 indicates that variance of number of donations is the function of the average number of donations and the repeat donor population size. The following proposition suggests that the higher the average of number of donations by repeat donors, the higher the variance. In larger blood supply chains, higher variability of donated blood by repeat donors is expected.

**Proposition 2.** The variance of the number of donations from repeat donors ( $Var(L)$ ) is increasing function of the average number of donations from the repeat donors

$(\mathbb{E}(L))$ . (Proof in Appendix A).

The following proposition investigates the impact of improving  $p$  on blood supply chain planning. It shows if the population of repeat donors would remain unchanged and the recurrence probability ( $p$ ) is improved, both average and variance of collected blood from repeat donors increase. However, to reach a targeted average daily donations by repeat donors, improving  $p$  of the existing repeat donors would be more effective than recruiting additional repeat donors (increasing repeat donor population). The justification behind this proposition is that variability of the collected blood by repeat donors is lower where the recurrence probability is improved compared to the situation that the decision makers tend to recruit additional repeat donors.

**Proposition 3.** By improving  $p_i$ 's, both the required population of repeat donors ( $N$ ) to achieve the same average number of donations ( $\mathbb{E}(L)$ ), and the variability of the number of donations from repeat donors ( $Var(L)$ ) decrease (Proof in Appendix A).

The following remark can be derived from Proposition 3.

**Remark 1.** Implementing policies to increase the recurrency of repeat donors results in requiring fewer repeat donors and will improve supply chain planning due to lower variation in the quantity of donated blood per day.

Blood services keep track of blood donors in particular repeat donors and use various initiatives to improve their recurrence probability ( $p$ ). The following proposition investigates which donors should be targeted for recurrence probability improvement.

**Proposition 4.** Given that  $p_i$  and  $p_j$  are the recurrence probability of donors  $i$  and  $j$ , where  $p_i < p_j$ , increasing  $p_i$  to  $p_i + \Delta$  is more efficient to improve  $\mathbb{E}(L)$  than increasing  $p_j$  to  $p_j + \Delta$ . Therefore, Increasing  $p$  to  $p + \Delta$  in donors with less recurrence probability is more efficient (Proof in Appendix A).

**Remark 2.** To increase blood donations, a blood bank is better off improving the recurrence probability ( $p$ ) of donors with small  $p$ . Therefore, it is recommended that blood services focus their resources on improving  $p$  of donors with lower recurrence probabilities.

The following proposition finds the mean and variance of the number of donation per day when  $p \sim \text{Beta}(\alpha, \beta)$  and shows that the number of donations per day follows a normal distribution. We chose Beta distribution as it has a support of  $[0, 1]$  that is needed for  $p$ . Furthermore, Beta distribution is versatile in shape and can be fitted to both right and left skewed data.

**Proposition 5.** Number of blood donations ( $L$ ) from repeat donors with a limited but large population size ( $N$ ) when the probability of giving donation in a particular day from an available repeat donor is a random variable with Beta distribution (i.e.  $p \sim \text{Beta}(\alpha, \beta)$ ), is approximately normal:

$$L \sim \text{Normal}(\mu, \sigma^2)$$

where

$$\mu = N \alpha \Gamma(\alpha + \beta) \widetilde{{}_2F_1}(1, \alpha + 1; \alpha + \beta + 1; -T) \quad (5)$$

and

$$\begin{aligned} \sigma^2 = N(\alpha \Gamma(\alpha + \beta) \widetilde{{}_2F_1}(1, \alpha + 1; \alpha + \beta + 1; -T) - \\ (\alpha \Gamma(\alpha + \beta) \widetilde{{}_2F_1}(1, \alpha + 1; \alpha + \beta + 1; -T))^2 \end{aligned} \quad (6)$$

where  $\widetilde{{}_2F_1}$  is the regularized Hyper-geometric function (Proof in Appendix A).

The following proposition outlines a methodology to estimate the average and

variance of the number of donations by repeat donors when the distribution of  $p$  is estimated by any function like  $\hat{f}(p)$  which is not necessarily *Beta*.

**Proposition 6.** Given  $\hat{f}(p)$  as an estimated distribution function of the probability of giving donation in a particular day ( $f(p)$ ), a random sample  $\mathbf{p}_1, \dots, \mathbf{p}_N$  can be generated from  $\hat{f}(p)$  to estimate the average and variance of number of blood donations from repeat donors as follows (Proof in Appendix A).

$$\hat{\mu}_L = \sum_{i=1}^N \frac{\mathbf{p}_i}{1 + T\mathbf{p}_i} \quad (7)$$

and

$$\hat{\sigma}_L^2 = \hat{\mu}_L - \frac{\hat{\mu}_L^2}{N} \quad (8)$$

In summary, by using the Markov chain method, the expected value and variance of daily donations for repeat donors has been mathematically modelled. Additionally, it has indicated that the variance of the number of donations from repeat donors is an increasing function of the expected value. An insight has been also derived from the mathematical model as higher number of donations can be obtained if decision makers put more efforts on improving the recurrence probability of the repeat donors with lower  $p$ . Besides, given that the recurrence probability follows the *Beta* distribution, mean and standard deviation of number of donations have been estimated. Finally, it has been explained that if distribution of the recurrence probability is obtained from the actual records of donations, how the average and variance of number of donations can be estimated from the the random samples.

#### 4. Modelling of Blood Donations in the Aftermath of a Disaster

In the previous section, the distribution of daily blood donation was investigated. The next step is to test the impact of an urgent call on the continuity of blood supply chain.



During an urgent call, since the number of donation increases, significant number of repeat donors will stay in the lockout period afterwards, and thus a shortage of blood donation is expected in the aftermath of a disaster.

#### 4.1. *Implications of Urgent Calls on the Expected Number of Repeat Donors in Phase III*

The literature review demonstrated that the first time donors who donated during an urgent call for blood will not become repeat donors. In other words, the rate of donations from the first time donors in the period before a disaster (phase I) and in the period after the end of an urgent call (phase III) are the same. Consequently, the blood demand and supply equilibrium mainly relies on repeat donors in phase III. Therefore, in the following, we only consider the increase in collections from repeat donors during urgent calls for blood that will cause a decline in donated blood in phase III due to unavailability of a fraction of repeat donors. Let  $E(L_{uc})$  be the expected number of repeat donors that respond when an urgent call for blood is placed. So:

$$\mathbb{E}(L_{uc}) = \mathbb{E}(L) + \gamma \quad (9)$$

where  $\gamma = \theta \times \mathbb{E}(L)$  and  $\theta$  is the multiplicative coefficient of increased donation. Equation 9 is aligned with what have been observed in several past calls for blood after disasters. For example, in the Bam earthquake it was reported that  $\theta \approx 6$  [Kasraian \(2010\)](#) and in the San Francisco earthquake,  $\theta$  was 2 ([Busch et al. 1991](#)). So, for the  $T$  days following the increase in donations it can be shown that the population of repeat (regular) donors would be:

$$N' = N - \gamma l \quad (10)$$

where  $l$  is the number of days over which the call for blood was responded. In the Black Saturday bushfires  $l$  was 10. Then  $N$  should be replaced by  $N'$  in Equation 7 to

obtain the average daily number of donations by repeat donors in phase III. If  $p$  can be fitted to a *Beta* distribution, using Equation 10 and multiplying its both sides by  $\alpha \Gamma(\alpha + \beta) {}_2\widetilde{F}_1(1, \alpha + 1; \alpha + \beta + 1; -T)$  and considering Equation 5, it can be concluded that the expected number of repeat donors that will donate after an urgent call for blood is given by:

$$\mathbb{E}(L') = \mathbb{E}(L) - \gamma l \alpha \Gamma(\alpha + \beta) {}_2\widetilde{F}_1(1, \alpha + 1; \alpha + \beta + 1; -T) \quad (11)$$

The blood supply chain must cope until the repeat donors that responded to the urgent call become eligible to donate again. From Equation 11, it can be concluded that the blood supply is reduced by  $k$  units per day in the first  $T - (l - 1)$  days of phase III where:

$$k = \gamma l \alpha \Gamma(\alpha + \beta) {}_2\widetilde{F}_1(1, \alpha + 1; \alpha + \beta + 1; -T) \quad (12)$$

To clarify how a blood supply chain reacts to an urgent call for blood in the short and long term, we review the three phases highlighted in the introduction section. In phase I or the steady state, there are on average  $\mathbb{E}(L)$  number of donations received on a daily basis from the repeat donors. If an urgent call arises for  $l$  days (phase II), the expected value of number of daily donations is increased to  $\mathbb{E}(L')$  by the multiplicative coefficient of  $\theta$  for  $l$  days. Given the lockout period ( $T$ ), blood supply chain suffers from a drop in supply ( $k$  units per day) for  $T - (l - 1)$  days. Subsequent to this period, the supply of blood by repeat donors recovers by the rate of  $\gamma$  for the next  $l - 1$  days. Phase III includes both periods of  $T - (l - 1)$  days and following  $l - 1$  days. In other words, once an urgent call for blood ends, the supply returns to the normal rate after  $T$  days. Then, the blood supply chain returns to its steady state equilibrium (phase I). The total decline in blood collection in phase III (denoted by  $\delta$ ) is obtained from Equation 13 given that  $p$  follows a *Beta* distribution. Note that  $k$ , obtained from Equation 12, is the estimated blood donation shortfall per day in

the first  $T - (l - 1)$  days of phase III. Then, in the last  $l - 1$  days of phase III, repeat donors who responded to the urgent call for blood start to return to the available donor pool.

$$\delta = (T - (l - 1))k + \sum_{i=1}^{l-1} (l - i) \frac{k}{l} \quad (13)$$

If a *Beta* distribution cannot be fit to  $p$ , then the total decline in blood collection in phase III (denoted by  $\delta$ ) is obtained from Equation 14.  $\mathbf{p}_i$ 's are random value generated from  $\hat{f}(p)$  that is the estimated distribution of  $p$ .

$$\delta = (T - (l - 1)) \sum_{i=N-l\gamma}^N \frac{\mathbf{p}_i}{1 + T\mathbf{p}_i} + \sum_{j=1}^{l-1} \sum_{i=N-j\gamma}^N \frac{\mathbf{p}_i}{1 + T\mathbf{p}_i} \quad (14)$$

#### 4.2. Estimating the Parameters of Recurrence Probability Distribution

From a practical perspective, it is crucial to understand how to estimate  $\hat{f}(p)$  from the historical records of blood donations of repeat donors. As explained earlier, the recurrence probability is the probability that an eligible (not in the lockout period) repeat donor gives blood. We also know this probability is different for each individual repeat donor. From the historical data of repeat donors' donations (sample size  $n$ ), we can estimate  $\hat{p}_i$  of each donor as:

$$\hat{p}_i = \frac{\text{number of donations by the donor } i^{\text{th}} \text{ in the period of given data}}{\text{number of days that donor } i^{\text{th}} \text{ was eligible for donation}} \quad (15)$$

Then, various distributions including empirical distribution can be used to obtain the best fitted distribution on  $\hat{p}_i$ 's. In case the *Beta* distribution, the population parameters ( $\alpha$  and  $\beta$ ) can be estimated from the sample statistics ( $\hat{\mu}_p$  and  $\hat{\sigma}_p^2$ ) using the maximum likelihood method or the method of moments. Applying the method of moments,  $\alpha$  and  $\beta$  are estimated as:

$$\hat{\alpha} = \hat{\mu}_p \left( \frac{\hat{\mu}_p(1 - \hat{\mu}_p)}{\hat{\sigma}_p^2} - 1 \right) \quad (16)$$

$$\hat{\beta} = (1 - \hat{\mu}_p) \left( \frac{\hat{\mu}_p(1 - \hat{\mu}_p)}{\hat{\sigma}_p^2} - 1 \right) \quad (17)$$

where  $\hat{\mu}_p$  and  $\hat{\sigma}_p^2$  are the sample mean and the sample variance of  $\hat{p}$  respectively.

In this section, we have discussed the additional increase in supply when a disaster strikes and an urgent call for blood is made. A mathematical model has been developed to estimate the decline in donations in the aftermath of a disaster since a number of repeat donors are stuck in the lockout period. It has also shown how the parameters of the *Beta* distribution ( $\alpha$  and  $\beta$ ) can be estimated from the actual records of donations.

## 5. Computational Experiments and Sensitivity Analysis

In this section numerical experiments are conducted to demonstrate the magnitude of blood collection shortfall in phase III as a result of an urgent call for blood in phase II. Additionally, a sensitivity analysis is carried out to analyse the impact of length of lockout period and parameters of the recurrence probability distribution on the anticipated decline in blood collection in phase III.

### 5.1. Numerical Experiments

Let us set  $N = 100,000$ ,  $T = 84$  and  $p_i \sim \text{Beta}(0.05, 1)$ . Considering the mean and variance of the Beta distribution, the mean and variance of the probability for a repeat donor to donate blood on any given day that they are available for donation (recurrence probability) are 0.0476 and 0.0221 respectively. In this case, according to Equation 5 in steady state period (phase I) on average 233.37 blood units per day are collected from the repeat donors population. The details and results of a simulation study to

generate the distribution of donated blood by repeat donor are presented in Appendix B.

The steady state blood supply chain is disturbed once a disaster strikes (phase II) and an urgent call may occur to make up for the the expected surge in demand for blood. The total reduction in collections in phase III ( $\delta$ ) obtained from Equation 13 for various values of  $\theta$  and  $l$  are presented in Figure 3.

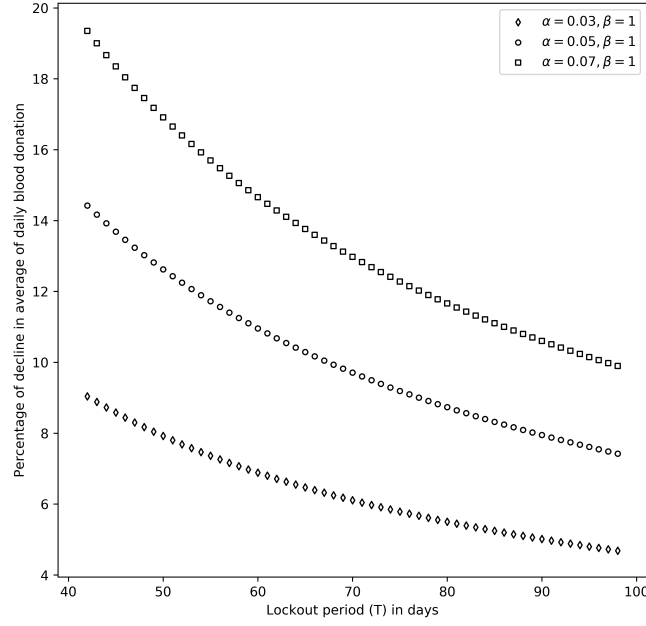
To illustrate the estimated decline in phase III, consider an 8-fold increase ( $\theta = 8$ ) in the number of donations over a period of 7 days ( $l = 7$ ). The expected number of additional donations from repeat donors would be around 13,069 (note that the average number of donations from repeat donors in a day was 233.37, therefore,  $233.37 \times 8 \times 7 = 13,068.72$ ). This will in turn cause a reduction in collection from repeat donors by 2470 units during phase III (30.5 units per day for 78 days, followed by a reduction of 26.14, 21.78, 17.42, 13.07, 8.71, 4.36 units for the next 6 days prior to returning the blood chain to its steady state). The intuition behind the decline of blood donations in the aftermath of an urgent call is that when a higher number of repeat donors give blood during phase II, they will become ineligible to donate again for the duration of lockout period ( $T$ ). Figure 3 illustrates that for higher values of  $\theta$  and  $l$ , the magnitude of decline in the number of donations after the urgent call for blood is higher. Put simply, increasing  $l$  and/or  $\theta$  leads to higher number of donations and thus more repeat donors will be put into lockout.

Decision makers in the blood sector would benefit from the analysis on how the multiplicative coefficient of blood donation ( $\theta$ ) and the length of urgent calls for blood campaign might moderate the severity of decline in the population of eligible donors in the aftermath of an urgent call.

## 5.2. Sensitivity Analysis

From the mathematical model, it is clear that the lockout period ( $T$ ) influences the decline in blood donation after an urgent call arises. The lockout period is determined by the national blood authorities in a country. For example, in Australia, the lockout period is 84 days (Australian Red Cross Blood Service 2018), whereas donors should wait for at least 56 days (8 weeks) to be eligible to donate again in some other coun-





**Figure 4.** Sensitivity analysis: the impact of lockdown period and Beta distribution's alpha parameter on the percentage of decline in average of daily blood donation. The y-axis shows the the percentage of decline in the average daily blood donations by the repeat donors in the next  $T - 6$  days after a call for blood resulted in a 6-fold increase in blood collection over 6 days.

the average decline in blood donation lower in the post disaster stage. For example, considering the fixed values of  $\alpha = 0.05$ ,  $\beta = 1$ ,  $l = 6$ , and  $\theta = 6$ , given the lockdown period in Australia (84 days), 8.4% decline in blood donation is expected, whereas the lower lockdown period in United States (56 days) leads to 11.6% decline in phase III. From a policy optimisation perspective, having a shorter lockdown period means a donor returns to the available pool sooner, thus more repeat donors are available for a given day. This is desirable for the blood service authorities as it reduces the required panel size of repeat donors that the blood service needs to maintain, and consequently reduces the variability of daily blood collection from repeat donors (see Equation 4). It also increases the number of repeat donors who respond to an urgent call for blood and magnifies the decline in blood collection in phase III. This insight makes decision makers aware that if the blood supply chain operates based on the lower values of lockdown period, the impact of an urgent call is more significant.

The second outcome of sensitivity analysis is the non-linear relationship between the

lockout period length and rate of daily decline of blood donation in phase III. As shown in Figure 4, the marginal rate of decreasing the blood donation is higher for lower values of lockout period. In other words, if one of the drivers to update the donation policy is to increase the lockout period in order to minimise the negative effect of an urgent call, it is necessary to consider that a single unit (day) decrease of the lockout period when the system operates in lower values of lockout period has more significant impact on mitigating the negative implication of an urgent call compared to the circumstances where higher values of lockout period are in place.

With respect to the sensitivity analysis of the selected distribution for the recurrence probability, considering the expected value of the *Beta* distribution, a higher value of  $\alpha$  results in increasing the average number of repeat donors. From the mathematical model, if more repeat donors give blood in the phase II, the supply chain would further suffer from a reduction in the population of available repeat donors in phase III. Therefore, as shown in Figure 4, the average decline in daily blood donation in the post disaster phase is higher for higher values of  $\alpha$ .

The computational results highlighted the magnitude of decline in the number of blood donations in the aftermath of an urgent call. It has been also shown how this rate of decline responds to various values of  $l$  and  $\theta$ . Sensitivity analysis has indicated that lower values of the lockout period ( $T$ ) intensifies the negative impact of an urgent call over-supply in the post disaster stage. Therefore, policy designers should be wary of significant decline in blood donation during phase III when the blood supply chain has a shorter lockout period. In addition, higher values of  $\alpha$  of *Beta* distribution leads to higher number of donations during a disaster and consequently to a higher post disaster decline in donations.

## 6. Urgent Call for Blood Policy Amendment

Subsequent to the above analysis, the remaining question is "*what action is the required by the blood services to avert the reduction of collected blood in phase III?*". Initially, the blood service needs to have an effective mechanism (e.g. a decision support model trained based on historical data) to estimate the additional units of blood required in



disaster events. Then, the proposed model adjusts the estimated demand to properly design a call for blood campaign to not only respond to the current surge in demand but to also consider the possibility of a decline in blood collection in phase III.

A remedy to the anticipated drop in blood collection in phase III is to over-collect blood in phase II. Considering the significance of a disaster, let  $D_s$  denote the total number of required blood units when a disaster strikes. The proposed model predicts the total decline in blood donation from repeat donors in phase III (see Equations 13 and 14). Given the estimation of a surge in demand ( $D_s$ ) in phase II and the total drop of blood donation from repeat donors in phase III ( $\delta$ ), urgent call for blood needs to be designed to cover the surge in demand in phase II as well as the anticipated drop in blood supply in phase III. The total blood units required to be collected during the urgent call for blood campaign ( $D_{uc}$ ) is obtained from Equation 18.

$$D_{uc} = D_s + \delta \quad (18)$$

The excess collection ( $\delta$ ) slightly increases the age of blood at transfusion since blood demands are fulfilled from the oldest items in the inventory (exhibiting a first in first out (FIFO) fashion). It means since day 1 of phase III, the over-collected blood is used and the length between the collection date and transfusion date of blood units collected in phase III increases. Blood services aim to supply fresh blood units as much as possible, hence, they have some policies in place (e.g. following up and postponing donor appointments) to moderate the age of blood at transfusion. These policies can be used after phase III once the blood supply chain returned to its steady state to revert the age of the transfused blood units to its target level.

In urgent calls for blood, although many of available donors (eligible at time of the call for blood) will respond, a big portion of collected blood is collected from first time donors who do not necessarily turn to be repeat donors in future (Tran et al. 2010), hence, majority of excess collection naturally will be collected from first time donors that will not affect the blood supply chain resilience in phase III and onward. However, the blood service may opt to collect the excess blood to avoid the decline in phase III ( $\delta$ ) from first time donors once sufficient blood required for the disaster

relief ( $D_s$ ) was collected.

It is worth noting that the accuracy of estimation of the surge in blood demanded during a disaster is highly crucial. An underestimation may jeopardise the life of affected people and put further pressure on the blood service running emergency campaigns. Conversely, collecting more than the surge in demand and the anticipated supply shortfall in phase III, will lead to an increase of the age of blood at transfusion and outdated blood units.

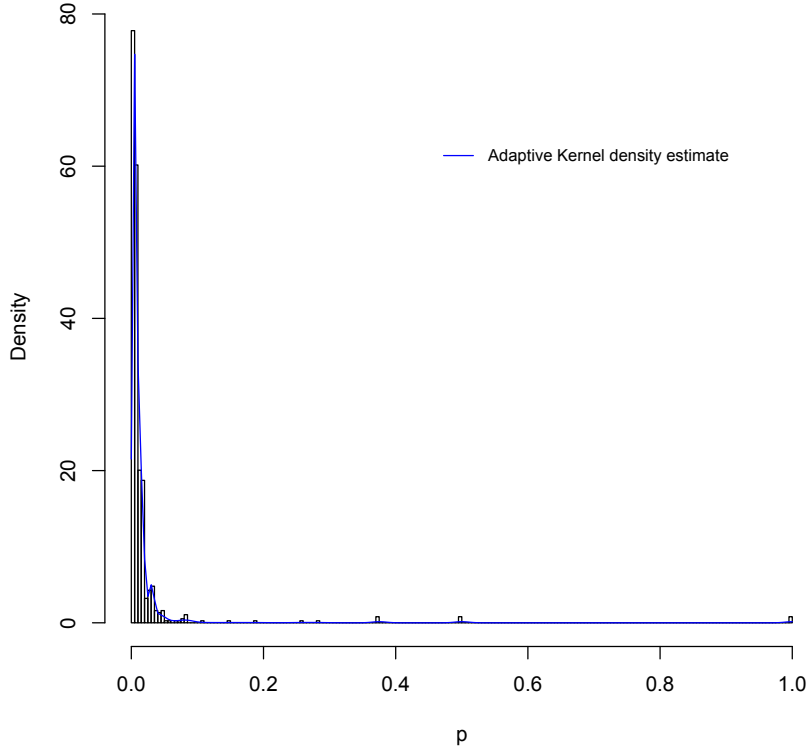
## 7. A Case Study

In this section, we use available real data of 748 repeat donors <sup>1</sup> (Yeh, Yang, and Ting 2009) to investigate the impact of calls for blood in the post disaster phase. The number of donations and the time that the donations are made (the time since the first donation) are available for each donor. The  $p$  for each donor by setting  $T = 56$  is estimated from Equation 15 providing 748 values of  $p$ . Using this data, the average and standard deviation of  $p$  are estimated as 0.0186 and 0.0758 respectively. We attempted to fit *Beta* and *Gamma* distributions to  $p$ , however, the p-value of goodness of fit test was very low (less than 0.0001) indicating these distributions cannot be used for this data. Thus, we used a non-parametric approach to estimate  $f(p)$ . Figure 5 shows the histogram of the data and a Gaussian adaptive kernel density estimate of  $f(p)$  (Silverman 1986, p. 100–104).

The average daily donations of blood by a repeat donor population of 100000 is 547.35 units in the steady state condition. To evaluate the impact of a call for blood lasting for 5 days which intensifies the blood donations 6 times of the donations in a normal day, we draw a sample of size 100000 from the empirical distribution and use Equation 7 where  $N$  is replaced by  $N'$  obtained from Equation 10 with  $l = 5$  and  $\theta = 6$  to estimate the average daily donations by the repeat donors in phase III. The average daily donations by the repeat donors will fall to 461 which is a significant impact on the blood supply chain. The impact of such decline will be

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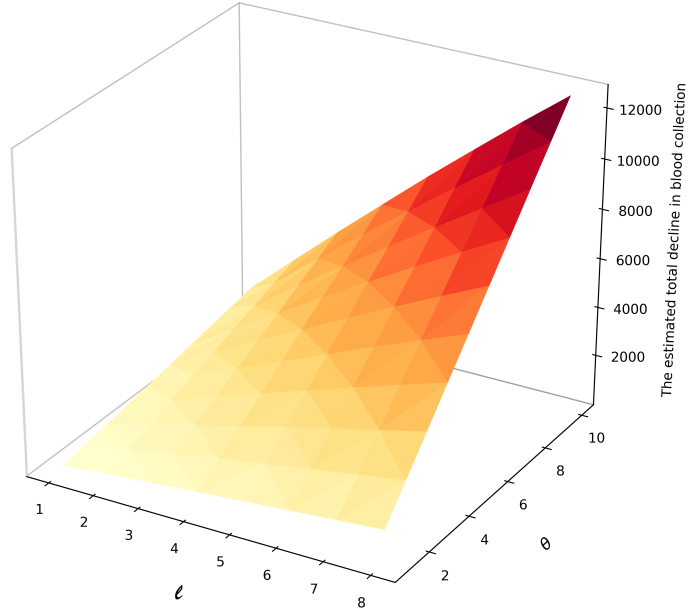
<sup>1</sup><https://archive.ics.uci.edu/ml/machine-learning-databases/blood-transfusion/transfusion.names>



**Figure 5.** The histogram and the Gaussian adaptive kernel density estimate of  $f(p)$  using the real data.

more serious in O negative blood type as its compatibility with other blood types increases its demand even further and ensuring a balance in its supply and demand is highly challenging even in a steady state situation. According to the proposed policy amendments, to remedy the expected collection decline in phase III, the blood service requires to over-collect 4869 units (obtained from Equation 14) more than what is required to respond to the disaster during an urgent call for blood. The blood service needs to alert the collection centres to cease collecting blood from repeat donors by postponing their appointments once sufficient blood to meet the surge in demand has been collected, and to continue to collect the required additional blood from first time donors to prevent shortages in phase III.

Figure 6 shows the total reduction of blood donations by the repeat donors in the next 56 days after a call for blood with different intensities and duration.



**Figure 6.** Case study: the average of total decline in blood donation during phase III ( $T = 56$  days,  $\theta = 1$  to 10,  $l = 1$  to 8).

Reviewing this case has revealed that subsequent to the over-supply of blood as a response to an urgent call, a significant decline in number of donations occurs. With respect to the lockout period of the case ( $T = 56$ ), it has been observed that the severity of post disaster decline of blood donations is higher compared to situation whether supply chain operates based on higher value of lockout period (e.g. Australia,  $T = 84$ ).

## 8. Conclusion

Previous studies have shown that new donors that respond to an urgent call for blood do not generally go on to become repeat donors. The existing population of repeat donors will also respond to the emergency but, as has been shown, their donations will be reduced for about 3 months in phase III as a direct consequence. The blood

supply chain must be able to wear this reduction. Additionally, blood types in high demand, such as O negative, would experience greater stress as these can be transfused into compatible patients with a different blood type. If all collected blood units are used during the emergency period, the blood supply chain will face a significantly increased risk of shortfalls over the following  $T$  days (e.g 84 days). The presented analysis quantified the expected shortfall of the blood supply in phase III as a consequence of an urgent call for blood and pointed out that such decline can be quite significant and may undermine the resilience of the blood supply chain. Therefore, to prevent a shortfall in donations by repeat donors in phase III, it is recommended that blood services assiduously collect extra units (equivalent to the anticipated decline in blood collection in phase III obtained from this analysis) during the urgent call for blood campaign. Furthermore, once a sufficient quantity of blood units required to respond to a disaster or emergency situation has been collected, the blood service needs to register repeat donors who present as a response to a call for blood for later appointments and proceed with collecting the required extra units to address the decline in supply in phase III from first time donors.

Due to the significant costs of over/under collection during an emergency and consequent negative impacts on the balance of blood supply chain, further study to determine better estimates of the quantity of blood donations needed in emergencies are necessary. These estimates will help to mitigate both the level of reduction (possibility of shortages) and quantity of outdates arising in response to an urgent call for blood. Even a detailed study on the most needed blood type during an emergency period considering the affected people would better inform an urgent call for blood.

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## Appendix A. Proofs

***Proof. Proposition 1.***

$Var(L)$  is expressed as:

$$Var(L) = Var(\mathbb{E}(L|p_i, s)) + \mathbb{E}(Var(L|p_i, s))$$

The first term in the above formula is expanded as:

$$Var(\mathbb{E}(L|p_i, s)) = \mathbb{E}(L^2|p_i, s) - \mathbb{E}(L|p_i, s)^2$$

Subsequently,

$$Var(\mathbb{E}(L|p_i, s)) = N \times ( \mathbb{E}((\frac{p_i}{1+Tp_i})^2) - \mathbb{E}(\frac{p_i}{1+Tp_i})^2 )$$

Also, as  $L$  given  $p$ 's has a Poisson binomial distribution and the variance of Poisson binomial distribution is  $\sum_{i=1}^N \frac{p_i}{1+Tp_i} (1 - \frac{p_i}{1+Tp_i})$ , the second term of the variance formula is simplified as:

$$\mathbb{E}(Var(L|p_i, s)) = \mathbb{E}(\sum_{i=1}^N \frac{p_i}{1+Tp_i} (1 - \frac{p_i}{1+Tp_i})) = N \times (\mathbb{E}(\frac{p_i}{1+Tp_i}) - \mathbb{E}((\frac{p_i}{1+Tp_i})^2))$$

Substituting the expanded expressions of the two terms of  $Var(L)$ :

$$\begin{aligned}
\sigma^2 &= Var(L) \\
&= N \times (\mathbb{E}(\frac{p_i}{1 + Tp_i}) - \mathbb{E}(\frac{p_i}{1 + Tp_i})^2) \\
&= \mathbb{E}(L) - \frac{\mathbb{E}(L)^2}{N}
\end{aligned}$$

□

***Proof. Proposition 2.***

From Proposition.1 we have:

$$Var(L) = \mathbb{E}(L) - \frac{\mathbb{E}(L)^2}{N}$$

Then,

$$\frac{\partial Var(L)}{\partial \mathbb{E}(L)} = 1 - 2 \times \frac{\mathbb{E}(L)}{N}$$

Now lets obtain the highest value that  $\mathbb{E}(L)$  for a given repeat donor population  $N$ :

$$\max \mathbb{E}(L) = \max_{p_i, s, f(p_i)} \int_0^1 \sum_{i=1}^N \frac{p_i f(p_i)}{1 + Tp_i} dp_i$$

As donors are independent we can maximise it for a given individual donor as:

$$\max_{p_i, f(p_i)} \int_0^1 \frac{p_i f(p_i)}{1 + Tp_i} dp_i$$

Now lets first investigate the highest value of  $z = \frac{p_i}{1+Tp_i}$ . The first order differentiation of  $z$  is:

$$\frac{\partial z}{\partial p_i} = \frac{1}{(1+Tp_i)^2}$$

Since the first order differentiation of  $z$  is always positive, the highest value of  $z$  is obtained for the highest possible value of  $p_i$  that is 1 and therefore the highest value of  $z$  can be  $\frac{1}{T+1}$ . Now by allocating the highest probability of  $p_i$  in  $f(p_i)$  to  $p_i = \frac{1}{T+1}$  we get:

$$\max_{p_i, f(p_i)} \int_0^1 \frac{p_i f(p_i)}{1+Tp_i} dp_i = \frac{1}{T+1}$$

Thus:

$$\max \mathbb{E}(L) = \frac{N}{T+1}$$

$\frac{\partial Var(L)}{\partial \mathbb{E}(L)} = 1 - 2 \times \frac{\mathbb{E}(L)}{N}$  for any value of  $\mathbb{E}(L)$  between 0 and  $\frac{N}{T+1}$  is positive. Therefore,  $Var(L)$  always increases by increasing  $\mathbb{E}(L)$  that completes the proof.  $\square$

***Proof. Proposition 3.***

In part of the proof of Proposition 2, we showed that the first order differentiation of  $z = \frac{p_i}{1+Tp_i}$  is always positive. Hence by increasing  $p_i$  the average number of donation by repeat donors increases or put it differently the previous average number of donations per day can be achieved from fewer repeat donors for example  $N_0$ . By exploring the formula of  $Var(L)$  by fixing  $\mathbb{E}(L)$  and changing  $N$  to  $N_0$ ,  $Var(L)$  decreases.  $\square$

**Proof. Proposition 4.**

The probability of a donation in a given day from a repeat donor with recurrence probability  $p$  is  $z = \frac{p}{1+Tp} \cdot \frac{\partial z}{\partial p_i} = \frac{1}{(1+Tp_i)^2}$  is always positive and it is a decreasing function of  $p$  indicating for  $p_1 < p_2$  we have  $z(p_1 + \Delta) - z(p_1) > z(p_2 + \Delta) - z(p_2)$ . This completes the proof. □

**Proof. Proposition 5.**

Expected Value

$\frac{p_i}{1+Tp_i}$  are independent variables, and when  $N$  is large  $\mathbb{E}(\mathbb{E}(L|p's))$  is  $\sum_{i=1}^N \frac{p_i}{1+Tp_i}$ . Therefore, the mean of  $L$  is obtained as:

$$\begin{aligned}
\mathbb{E}(L) &= \mathbb{E}(\mathbb{E}(L|p's)) \\
&= \int_0^1 \sum_{i=1}^N \frac{p_i f(p_i)}{1+Tp_i} dp_i \\
&= \sum_{i=1}^N \int_0^1 \frac{p_i \Gamma(\alpha + \beta) p_i^{(\alpha-1)} (1-p_i)^{(\beta-1)}}{(1+Tp_i) \Gamma(\alpha) \Gamma(\beta)} dp_i \\
&= \sum_{i=1}^N \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha) \Gamma(\beta)} \int_0^1 \frac{p_i^\alpha (1-p_i)^{(\beta-1)}}{(1+Tp_i)} dp_i \\
&= \sum_{i=1}^N \frac{\alpha \Gamma(\alpha + \beta + 1)}{(\alpha + \beta) \Gamma(\alpha + 1) \Gamma(\beta)} \int_0^1 \frac{p_i^\alpha (1-p_i)^{(\beta-1)}}{(1+Tp_i)} dp_i \\
&= \sum_{i=1}^N \frac{\alpha \Gamma(\alpha + \beta)}{\Gamma(\alpha + 1) \Gamma(\beta)} \int_0^1 \frac{p_i^\alpha (1-p_i)^{(\beta-1)}}{(1+Tp_i)} dp_i
\end{aligned}$$

Recall that:

$$\widetilde{{}_2F_1}(a, b; c; z) = \frac{1}{\Gamma(b) \Gamma(c-b)} \int_0^1 \frac{t^{(b-1)} (1-t)^{(c-b-1)}}{(1-zt)^a} dt$$

is the Regularized Hypergeometric function. So:

$$\begin{aligned} \mathbb{E}(L) &= \sum_{i=1}^N \alpha \Gamma(\alpha + \beta) \widetilde{{}_2F_1}(1, \alpha + 1; \alpha + \beta + 1; -T) \\ &= N \alpha \Gamma(\alpha + \beta) \widetilde{{}_2F_1}(1, \alpha + 1; \alpha + \beta + 1; -T) \end{aligned}$$

### Standard Deviation

Substituting the expected value obtained above into the variance formula in Proposition.1:

$$\begin{aligned} \sigma^2 &= Var(L) \\ &= N \times (\mathbb{E}(\frac{p_i}{1+Tp_i}) - \mathbb{E}(\frac{p_i}{1+Tp_i}))^2 \\ &= N \times (\alpha \Gamma(\alpha + \beta) \widetilde{{}_2F_1}(1, \alpha + 1; \alpha + \beta + 1; -T) \\ &\quad - (\alpha \Gamma(\alpha + \beta) \widetilde{{}_2F_1}(1, \alpha + 1; \alpha + \beta + 1; -T))^2) \end{aligned}$$

When  $N$  is large (that is the case in blood donations), according to central limit theorem the distribution of sum of a large number of independent random variables approaches to Normal distribution. This completes the proof.  $\square$

***Proof. Proposition 6.***



$$\begin{aligned}
\mathbb{E}(L) &= \mathbb{E}(\mathbb{E}(L|p, s)) \\
&= \int_0^1 \sum_{i=1}^N \frac{p_i f(p_i)}{1 + T p_i} dp_i \\
&= \sum_{i=1}^N \int_0^1 \frac{p_i f(p_i)}{1 + T p_i} dp_i \\
&= \sum_{i=1}^N \mathbb{E}\left(\frac{p}{1 + T p}\right) = N \mathbb{E}\left(\frac{p}{1 + T p}\right)
\end{aligned}$$

Given a random sample of size  $N$  for  $p$ , (i.e.  $\mathbf{p}_1, \dots, \mathbf{p}_N$ ),  $\mathbb{E}(\frac{p}{1+Tp})$  is estimated by  $\frac{\sum_{i=1}^N \frac{\mathbf{p}_i}{1+T\mathbf{p}_i}}{N}$ , therefore:

$$\mathbb{E}(\hat{L}) = \hat{\mu} = N \frac{\sum_{i=1}^N \frac{\mathbf{p}_i}{1+T\mathbf{p}_i}}{N} = \sum_{i=1}^N \frac{\mathbf{p}_i}{1 + T\mathbf{p}_i}$$

Moreover, by substituting  $\hat{\mu}_L$  in  $Var(L)$  formula obtained in Proposition 1, the estimated variance of  $L$  is given by:

$$\hat{\sigma}_L^2 = \hat{\mu}_L - \frac{\hat{\mu}_L^2}{N}$$

□

## Appendix B. Simulation

Assuming there are 100,000 repeat donors, the same number of samples was generated using  $Beta(0.05, 1)$  to represent the recurrence probability  $(p_i, i = 1, 2, \dots, 100000)$  of each donor. The  $k^{th}$  value of this sample represents the probability of donating blood by repeat donor  $k$  on a day when they are eligible to donate.

Given that the probability a donor gives blood in a given day is estimated as the probability of the donor being available ( $\pi_A$ ) times the recurrence probability of the

repeat donor ( $p_i$ ), 1000 random values from Poisson Binomial distribution with parameters ( $p_1, p_2, \dots, p_{100000}$ ) were generated. Each of these 1000 values represents the total quantity of donated blood from repeat donors in a given day. Algorithm 1 presents the pseudo code of the simulation steps.

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**Algorithm 1:** The pseudo code of simulation algorithm used to generate the numbers of donations by repeat donors. The repeat donor population is assumed to be 100000 ( $N = 100000$ ) and  $T = 84$ .

---

**Initialization**

Generate 100000 random numbers from  $Beta(\alpha = 0.05, \beta = 1)$ . Name them as

$p_1, \dots, p_{100000}$ ;

**for**  $m=1$  **to** 1000 **do**

**for**  $k=1$  **to** 100000 **do**

$\zeta(k) \leftarrow$  Generate a random value from  $Bernoulli(\frac{p_i}{1+84p_i})$ ;

$L(m) = \sum_{j=1}^{100000} \zeta(j)$ ;

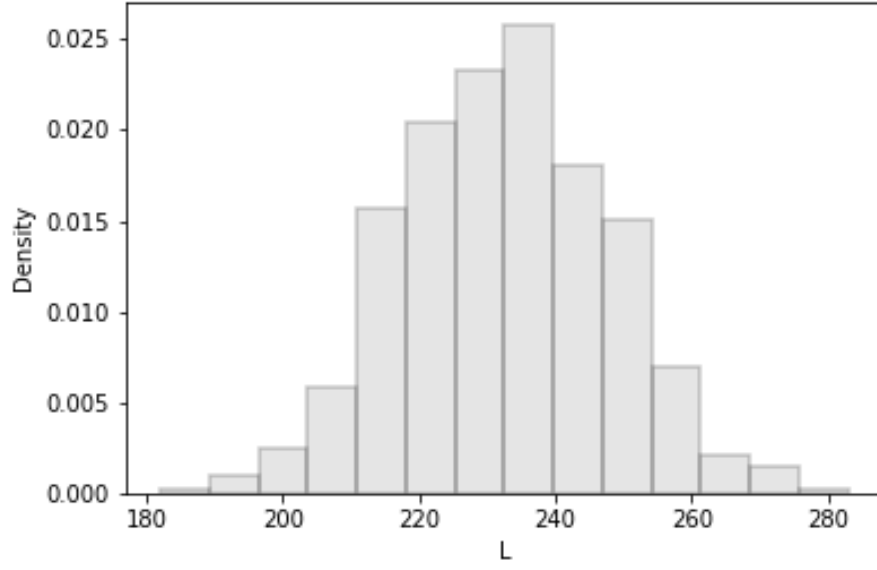
**Report**  $L$ ;  $L$  is a vector of size 1000 where each element represents the number of donations from repeat donors with population of 100000 in a day.

---

Recall from the model development section the expected value of Poisson Binomial distribution is  $\sum p_i$ . Figure B1 shows the density histogram of the number of donated blood by repeat donors obtained from this computational experiment. The average and standard deviation of number of donations from repeat donors in a given day over phase I obtained from the simulation experiment are 232.45 and 15.30 respectively while their corresponding values obtained from (5) and (6) are 233.37 and 15.26 which verifies the results.

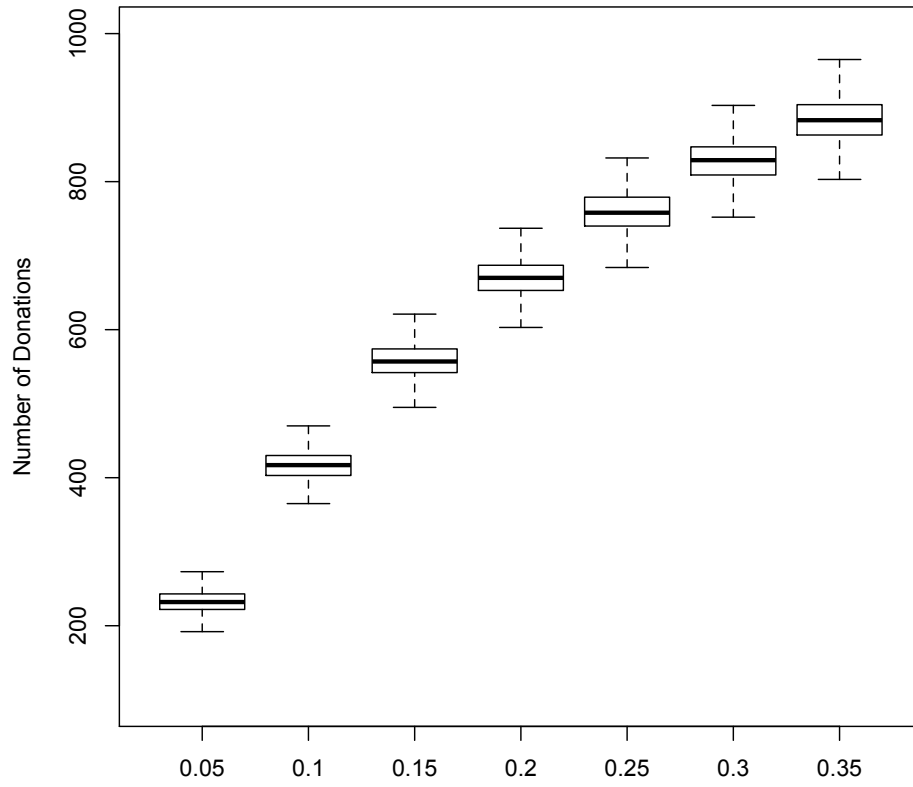
From a practical point of view, the planning units of blood centres could benefit from an understanding of the expected distribution of number of donated blood by repeat donors for future modelling practices. Therefore, it is worth looking into the probability distribution of number of donations in the phase I (steady state). The Anderson Darling normality test (on the obtained 1000 values from above simulation) results a p-value of 0.2592 that shows number of donations per day follows a normal distribution.

While it is possible to estimate the Beta distribution parameters ( $\alpha$  and  $\beta$ ) from the data using Equations 16 and 17, in this study  $\alpha$  and  $\beta$  have been selected as 0.05 and 1 to create a distribution with mean and variance that could plausibly represent what has been indicated in the literature. It is necessary to test the sensitivity of the



**Figure B1.** The distribution of the number of daily donations by repeat donors by 1000 replications when the population of repeat donors is 100000 and  $p \sim \text{Beta}(\alpha = 0.05, \beta = 1)$ .

number of blood donations per day for various values of  $\theta$ . This can be seen in Figure B2 which compares the number of donations from repeat donor population when  $\alpha$  changes. By increasing  $\alpha$ , both the average and variability of the number of donations increase. This also visually confirms Proposition 4.



**Figure B2.** The number of daily donations by repeat donors by 1000 replications when the population of repeat donors is 100000 and  $p \sim \text{Beta}(\alpha, \beta = 1)$  where  $\alpha$  (x-axis) changes from 0.05 to 0.3 (by the step of 0.05).